DFPD 93/TH/75
SISSA 93/183-A

Thermal Background Corrections to the Neutrino Electromagnetic Vertex in Models with Charged Scalar Bosons

A. Riotto[a,b,1]

[(a)] *International School for Advanced Studies, SISSA-ISAS*
Strada Costiera 11, 34014 Miramare, Trieste, Italy

[(b)] *Istituto Nazionale di Fisica Nucleare,*
Sezione di Padova, 35100 Padua, Italy.

Abstract

[1]Email:riotto@tsmi19.sissa.it

1. Introduction

It is well-known that the properties of neutrinos propagating through a medium differ form those in the vacuum; for instance, the vacuum energy-momentum relation for massless neutrinos $E_\nu = |\mathbf{p}_\nu|$, where E_ν is the energy and $|\mathbf{p}_\nu|$ is the magnitude of the momentum vector, is no longer valid in the medium [1]. The modification of the neutrino dispersion relation can be represented in terms of an index of refraction or an effective potential and arise, in the framework of finite-temperature field theory, from the temperature- and density-dependent corrections to the neutrino self-energy [2].

Of primary interest along this line is also the study of the electromagnetic interactions of neutrinos in a medium [3]. The dramatic enhancement of the radiative decay rate of neutrinos in an electron-rich medium has been investigated in ref. [4] and is due to the fact that the Glashow-Iliopoulos-Maiani (GIM) mechanism, which suppresses the radiative decay in vacuum, is inoperative for the matter contribution. Moreover, since the medium can introduce $\mathcal{CP}$ and $\mathcal{CPT}$ asymmetries in the effective potential interactions, Majorana neutrinos are allowed to get diagonal electric and magnetic dipole moments [3] in the Standard Model (SM) which are forbidden in the vacuum.

The great attention in the recent literature on the properties of neutrinos propagating in a medium has been motivated by the attractive suggestion that the solar neutrino problem [5] can be solved by the resonant oscillation mechanism [6]. Another possible explanation of the observed neutrino deficit from the sun is based on the interactions of solar neutrinos with the magnetic field of the outer layers of the sun. This requires large diagonal [7] and/or transition [8] magnetic moments for the electron neutrino, of order of $(10^{-11} - 10^{-10})\ \mu_B$, where μ_B is

the Bohr magneton. Moreover, light neutrinos possessing a magnetic moment of order of $10^{-12}\,\mu_B$ can play a role in many astrophysical phenomena such as the rapid cooling of degenerate stars and (if neutrinos are of the Dirac type) the emission of the collapse energy from the core of supernovae [9]. Also, a value of $\mu_\nu \sim 10^{-12}\,\mu_B$ is cosmologically acceptable for Dirac neutrinos [10].

Unfortunately in SM the magnetic moment is generated at the one-loop level and is extremely small because the only scales of the problems are the mass of the neutrino m_{ν_e} and the Fermi constant G_F. Indeed [11],

$$\mu_{\nu_e} = \frac{3\,e\,G_F}{8\,\pi^2\,\sqrt{2}}\,m_{\nu_e} \simeq 3\times 10^{-19}\,\left(\frac{m_{\nu_e}}{1\ \mathrm{eV}}\right)\,\mu_B. \tag{1}$$

It is clear that to get a magnetic moment of order of $10^{-12}\,\mu_B$ one has to invoke some new physics beyond the SM. Indeed, a large class of models [12, 13] which are able to provide large magnetic moments to neutrinos, have the common feature to posses new charged scalar bosons whose mass can be arbitrary [12] or fixed by the supersymmetric scale [13].

In the present work we give the results of detailed calculations of the background-dependent part of the $\nu\nu\gamma$ vertex when these new charges scalar bosons couple to leptons in a medium consisting of a gas of electrons. As usual, the electron gas is embedded in a uniform positive-ion background. However, the effect of ions is negligible in most circumstances [14].

For sake of concreteness, we have decided to perform all the calculations in a well-defined framework, namely the supersymmetric model with explicit breaking of R-parity [13, 15], where neutrinos are Majorana particles. The generalization to other models for both Majorana and Dirac neutrinos is straightforward [16].

We find that the magnetic (electric) dipole moment does not receive from the medium any significant enhancement, as suggested by Giunti *et al.* in ref. [3]

for the SM . However, a new chirality flipping, but helicity conserving, term is induced by the interactions with the thermal bath. This new term vanishes if the background is $\mathcal{CPT}$ symmetric and is associated to the longitudinal photon exchanged and therefore disappears in the vacuum. We estimate the contribution coming from this new term to the plasmon decay process $\gamma_{pl} \to \nu\nu$ [9], which is the primary source of the rapid cooling of degenerate stars, and show that it can be comparable to the contribution due to the vacuum magnetic moment.

We also show that, as in the case of SM [3], one-loop thermal corrections bring in an effective charge for Majorana neutrinos in a medium as well as a magnetic (electric) diagonal dipole moment which would not be allowed in the vacuum. Moreover, the effective potential receives a correction in presence of an external magnetic field.

The paper is organized as follows. In Section **2** we present the model we have adopted to illustrate our calculations. In Section **3** the calculations are described and general formulas for the form factors are given in terms of integrals over the electron-positron energy distribution. Some details of the calculations and the results in different limits are given in the Appendix. Then in Section **4** we estimate the plasmon decay rate contribution from the new terms arising in the medium. Section **5** presents our conclusions.

2. The model

The minimal supersymmetric standard model [17] with explicit R-parity breaking [15] via L-violation is described by the superpotential which, in addition to

the standard Yukawa couplings, involves the $\Delta L \neq 0$ couplings

$$f^{\Delta L \neq 0} = \frac{1}{2}\lambda_{ijk}\ [L_i, L_j]\, e^c_k + \lambda'_{ijk}\ L_i Q_j d^c_k, \tag{2}$$

where, i, j, k are generation indices, L, Q are the lepton and the quark left-handed doublets and e^c, d^c are (the charge conjugate of) the right-handed lepton and charge $-1/3$ quark singlets, respectively. The first term in eq. (2) gives rise to the Lagrangian

$$\mathcal{L}^{\Delta L \neq 0} = \lambda_{ijk}\left[\tilde{l}^j_L \bar{l}^k_R \nu^i_L + \left(\tilde{l}^k_R\right)^* \left(\bar{\nu}^i_L\right)^c l^j_L + \tilde{\nu}^i_L \bar{l}^k_R l^j_L - (i \leftrightarrow j)\right] + \text{h.c.}, \tag{3}$$

where $l^c = C\bar{l}^T$ means the charge conjugated of the fermion l, C being the charge conjugation matrix, and we have disregarded the second term in eq. (2) since we are interested in a medium consisting of electrons and positrons.

In the vacuum the couplings of eq. (3) give rise to neutrino masses and magnetic moments (after the insertion of a photon vertex in any charged internal line) through two different one-loop diagrams, see figure 1. In all the diagrams of figure 1 an helicity flip on the internal fermion line is necessary. As explicitly indicated, this also requires a mixing of the scalar leptons associated with the different chiralities. Any of the diagrams of figure 1 contribute to $m_{\nu_i\nu_j}$ and $\mu_{\nu_i\nu_j}$ as

$$m_{\nu_i\nu_j} \simeq \frac{\lambda_a\lambda_b}{16\pi^2}\, m\, \frac{\sin 2\theta_k}{2}\left(\frac{1}{m^2_{1k}} - \frac{1}{m^2_{2k}}\right), \tag{4}$$

$$\begin{aligned}\mu_{\nu_i\nu_j} &= \mu_B \frac{\lambda_a\lambda_b}{8\pi^2}\, m_e\, m \sin 2\theta_k \left\{\frac{1}{m^2_{1k}}\left[\ln\left(\frac{m^2_{1k}}{m^2}\right) - 1\right]\right.\\ &- \left.\frac{1}{m^2_{2k}}\left[\ln\left(\frac{m^2_{2k}}{m^2}\right) - 1\right]\right\},\end{aligned} \tag{5}$$

where m is the mass of the internal charged lepton, λ_a and λ_b are the appropriate couplings, and m_{1k}, m_{2k} and θ_k are the two mass eigenvalues and the mixing angle of the $\tilde{l}^k_L\tilde{l}^k_R$ mixing matrix, respectively.

If, for instance, we examine the $\nu_e\nu_\mu\gamma$ vertex, since the contribution to $\mu_{\nu_e\nu_\mu}$ turns out to be proportional to $m_{\nu_e\nu_\mu}$ and we require $m_{\nu_e\nu_\mu} < \mathcal{O}(10)$ eV, a strong bound on $\mu_{\nu_e\nu_\mu}$ is obtained, roughly $\mu_{\nu_e\nu_\mu} < \mathcal{O}(10^{-14})\mu_B$. To enhance the vacuum magnetic moment $\mu_{\nu_e\nu_\mu}$ to the astrophysically interesting value of $10^{-12}\mu_B$, one can follow ref. [13] and impose that the lepton number $L_e - L_\mu$ remains unbroken and that the Lagrangian, in the limit of vanishing Yukawa couplings, is symmetric under an $SU(2)_H$ horizontal symmetry acting on the first and the second generations. Under this assumption, the only terms which survive in eq. (2) are

$$f^{\Delta L\neq 0} = \lambda_{123} L_e L_\mu \tau^c \ + \ \lambda_{131}\left(L_e L_\tau e^c + L_\mu L_\tau \mu^c\right). \qquad (6)$$

The corresponding graphs giving rise to $m_{\nu_e\nu_\mu}$ and $\mu_{\nu_e\nu_\mu}$ are given in figure 2. Since under the horizontal symmetry $SU(2)_H$ the mass term $m_{\nu_e\nu_\mu}$ is odd, whereas $\mu_{\nu_e\nu_\mu}$ is even, diagrams 2a) and 2b) and 2c) and 2d) tend to cancel out and to sum up for $m_{\nu_e\nu_\mu}$ and $\mu_{\nu_e\nu_\mu}$, respectively. As a consequence, $\mu_{\nu_e\nu_\mu}$ is now no longer proportional to $m_{\nu_e\nu_\mu}$ and the value $\mu_{\nu_e\nu_\mu} \simeq 10^{-12}\mu_B$ can be achieved [13].

If we now consider a medium filled up with a gas of electrons, the Lagrangian which gives rise to the finite-temperature effective vertices involving ν_e, ν_μ and γ is

$$\mathcal{L} = \lambda_{131}\tilde{\tau}_L \bar{e}_R \nu_{eL} \ - \ \lambda_{123}\tilde{\tau}_R^* \left(\bar{\nu}_{\mu L}\right)^c e_L \ + \text{h.c.}. \qquad (7)$$

This will be our starting Lagrangian in the next Section[2].

[2] Even if we are focusing on a particular vertex, $\nu_e\nu_\mu\gamma$, in a particular model, we want to stress again that the structure of the form factors derived in the next Section are model-independent.

3. Calculation of the vertex functions in the medium

3.1 Chirality flipping terms

In this Subsection we calculate the contribution to the chirality flipping term $(\nu_{\mu L})^c\,\nu_{eL}\gamma$ in the medium assuming that the temperature is such that there are no charged scalar particles $\tilde{\tau}_L$ and $\tilde{\tau}_R$ in the background. Therefore, only the electron propagator has a background-dependent part and is given by

$$S_F(k) = (\not{k} + m_e)\left[\frac{1}{k^2 + m_e^2} + 2\pi i\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right)\right], \tag{8}$$

where

$$\eta(x) = \frac{\theta(x)}{\mathrm{e}^{\beta(x-\mu)}+1} + \frac{\theta(-x)}{\mathrm{e}^{-\beta(x-\mu)}+1}. \tag{9}$$

Here $\theta(x)$ is the unit step function, $1/\beta$ is the temperature, μ is the electron chemical potential and u^μ is the four-velocity of the center of mass of the medium.

The off-shell electromagnetic vertex function $\Gamma_\mu^{LR}\left(p_1,p_2,u\right)$ is defined in such a way that

$$\langle\nu_\mu\left(p_1\right)\left|J_\mu^{EM}(0)\right|\nu_e\left(p_2\right)\rangle \equiv \bar{u}_\mu\left(p_1\right)\Gamma_\mu^{LR}\left(p_1,p_2,u\right)u_e\left(p\right). \tag{10}$$

Note that in the vacuum the dependence on u^μ vanishes. The diagrams which enter the calculation of Γ_μ^{LR} are shown in figure 3.

Since the integrals involved in the calculations of Γ_μ^{LR} are cut off by the electron-positron distribution, the diagram 3b) gives a contribution to Γ_μ^{LR} suppressed by an extra power of $1/\tilde{m}^2$, $\tilde{m}$ being the typical supersymmetric mass, relative to the diagram 3a). Therefore, we neglect it.

With this preliminaries, we have to calculate the following quantity

$$-i\mathcal{G}_\mu^{LR} \quad = \quad e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \int \frac{d^4k}{(2\pi)^4}\, iS_F\left(k-q\right)\gamma_\mu iS_F\left(k\right)L$$

$$\times \quad \left[\frac{1}{(k-p)^2 - m_{23}^2} - \frac{1}{(k-p)^2 - m_{13}^2}\right], \tag{11}$$

where $L = (1-\gamma_5)/2$ is the left-handed chirality operator.

When the electron propagator from eq. (8) is plugged in eq. (11), several terms are produced beyond the standard vacuum term. The terms with two factors $\eta(k \cdot u)$ contribute only to the absorptive part of the amplitude (see, for instance, D'Olivo *et al.* in ref. [3] for further comments on this points). In this paper we will calculate only the dispersive part of the form factors. We also make the local approximation, *i.e.* neglect the momentum dependence of the heavy charged scalar bosons; hence $\mathcal{G}_\mu^{LR}$ reduces to

$$\begin{aligned}
\mathcal{G}_\mu^{LR} &= e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] \int \frac{d^4k}{(2\pi)^3} \left(\not{k} - \not{q} + m_e\right)\gamma_\mu \left(\not{k} + m_e\right) \\
&\times \left\{\frac{\delta\left[(k-q)^2 - m_e^2\right]}{k^2 - m_e^2}\eta\left[(k-q)\cdot u\right]\right. \\
&+ \left.\frac{\delta\left(k^2 - m_e^2\right)}{(k-q)^2 - m_e^2}\eta\left(k \cdot u\right)\right\} L.
\end{aligned} \tag{12}$$

Making the change of variable $k \to k + q$ in the first integral of eq. (12) we obtain

$$\begin{aligned}
\mathcal{G}_\mu^{LR} &= e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e \\
&\times \left(\mathcal{I}_\mu^1 + \mathcal{I}_\mu^2 + \mathcal{I}_\mu^3\right) L,
\end{aligned} \tag{13}$$

where we have defined

$$\mathcal{I}_\mu^1 = \int \frac{d^3k}{(2\pi)^3}\frac{(f_- - f_+)}{2E}\frac{4k^\mu q^2}{(q^2 + 2k\cdot q)(q^2 - 2k\cdot q)}, \tag{14}$$

$$\mathcal{I}_\mu^2 = \int \frac{d^3k}{(2\pi)^3}\frac{(f_- + f_+)}{2E}\frac{-2i\sigma_{\mu\nu}q^\nu\, q^2}{(q^2 + 2k\cdot q)(q^2 - 2k\cdot q)}, \tag{15}$$

$$\mathcal{I}_\mu^3 = \int \frac{d^3k}{(2\pi)^3}\frac{(f_- - f_+)}{2E}\frac{-4\,(k\cdot q)\ q_\mu}{(q^2 + 2k\cdot q)(q^2 - 2k\cdot q)}, \tag{16}$$

and $\sigma_{\mu\nu} = (i/2)\left[\gamma_\mu, \gamma_\nu\right]$, $k^\mu = (E, \mathbf{k})$, $E = \sqrt{|\mathbf{k}| + m_e^2}$, and we have introduced the electron and positron distributions

$$f \mp (k) = \frac{1}{\mathrm{e}^{\beta(k \cdot u \mp \mu) + 1}}. \tag{17}$$

Note that $q^\mu \mathcal{G}_\mu^{LR} = 0$ due to the electromagnetic gauge invariance.

If the initial Lagrangian (in the vacuum) of our model respects $\mathcal{CP}$ (so that we take all the λ's real), we can define the four-component self-conjugate states $\chi_a = \nu_a + \eta_a \left(\nu_a\right)^c$, where $\eta_a = \pm 1$ are the intrinsic $\mathcal{CP}$ parities of χ_a's. It is well-known that if χ_e and χ_μ have opposite (equal) $\mathcal{CP}$ parities, then their off-diagonal dipole (magnetic) moment vanishes [18]. Moreover, the correct expression for $\Gamma_\mu^{LR,Majorana}$ can be derived form expressions (13-16) once one remembers that χ_e and χ_μ are Majorana neutrinos, so that for each Feynman diagram there exists a second diagram in which all the internal particles are replaced by their charge conjugates [18]. A practical rule to derive $\Gamma_\mu^{LR,Majorana}$ is to treat neutrinos as Dirac particles and then add to $\Gamma_\mu^{LR,Dirac}$ its charge conjugate part

$$\begin{aligned}\Gamma_\mu^{LR,Majorana}\left(p_1, p_2, u\right) &= \Gamma_\mu^{LR,Dirac}\left(p_1, p_2, u\right) \\ &+ \eta_e \eta_\mu\, \gamma^0 \left[C\, \Gamma_\mu^{LR,Dirac}\left(-p_1, -p_2, u\right)\, C^{-1}\right]^* \gamma^0. \end{aligned}\tag{18}$$

If we now introduce the following tensors

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \tag{19}$$

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu, \tag{20}$$

and remind that, as a consequence of having taken the limit $\tilde{m} \to \infty$, the form factors depend only on q and not on p_1 and p_2 separately, the complete off-shell

electromagnetic vertex function reads

$$\Gamma_{\mu}^{LR,Majorana} = \left[F_1 \tilde{u}_{\mu}\left(L + \eta_e \eta_{\mu} R\right) + i\frac{F_2}{2}\left(1 - \eta_e \eta_{\mu}\right)\sigma_{\mu\nu}q^{\nu} + i\frac{F_2}{2}\sigma_{\mu\nu}\gamma_5\left(1 + \eta_e \eta_{\mu}\right)q^{\nu}\right], \tag{21}$$

where $R = (1 + \gamma_5)/2$ is the right-handed chirality operator and

$$\begin{aligned} F_1 &= e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e\, 2q^2 \\ &\times \int \frac{d^3k}{(2\pi)^2}\frac{(f_- - f_+)}{2E}\frac{(u \cdot k)}{(q^2 + 2k \cdot q)(q^2 - 2k \cdot q)}, \qquad (22) \\ F_2 &= -e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e 2q^2 \\ &\times \int \frac{d^3k}{(2\pi)^2}\frac{(f_- + f_+)}{2E}\frac{1}{(q^2 + 2k \cdot q)(q^2 - 2k \cdot q)}. \qquad (23) \end{aligned}$$

The convenience of presenting the form factors F_1 and F_2 in this form relies on the fact that F_1 and F_2 are scalars so that the integrations can be performed in the rest-frame of the medium defined by setting $u^{\mu} = (1, \mathbf{0})$. In th rest frame of the medium, we will denote the components of the four vector q^{μ} by

$$q^{\mu} = (\Omega, \boldsymbol{Q}), \tag{24}$$

where $\Omega \equiv q \cdot u$ and $|\boldsymbol{Q}| \equiv \sqrt{\Omega^2 - q^2}$ are manifestly scalar functions.
From expression (21) it is clear that the form factor F_2 can be regarded as an additional contribution to the magnetic (or electric) dipole moment. We note that, since the contribution to the magnetic (electric) dipole moment in the medium must be coherent with the neutrino propagation, it is necessary to take the limit $q^{\mu} \to 0$ in eq. (23). Depending on how one approaches the limit, eq. (23) can yield different results because of its divergent nature. However, when $q^{\mu} = 0$, the internal electron lines with four-momenta k and $k - q$ are on the mass shell, *i.e.* in the limit $q^{\mu} \to 0$, the diagram 3a) describe the process
$\nu_e e \to \nu_{\mu} e$ with a modification of the external electron lines by the

electromagnetic field. Therefore one can expect no enhancement in the medium for the magnetic (electric) dipole moment (see Giunti *et al.* in ref. [3]). Our result confirms this expectation.

The existence of the flipping term proportional to F_1 is a unique property of the thermal bath. It is due to the presence of the longitudinal photons which can couple to the internal electron without changing its helicity. Let us recall that in the vacuum, the magnetic (electric) moment flips both chirality and helicity (at the leading order), since the photon is purely transverse, while the vertex $F_1 \tilde{u}^\mu$ cannot change the helicity of the incoming neutrino. Moreover, if the chemical potential of the electron background is zero, then $f_+ = f_-$ and, therefore, $F_1 = 0$ for both Dirac and Majorana neutrinos. Indeed, if we start from a $\mathcal{CP}$ invariant Lagrangian in the vacuum and, for $\mu = 0$, the background is symmetric, F_1 must satisfy the relation

$$F_1\left(-p_1, -p_2, u\right) = -F_1\left(p_1, p_2, u\right), \tag{25}$$

independently of whether the neutrino is of the Dirac type or Majorana type. Since in our case F_1 is only a function of $p_1 - p_2$, eq. (25) implies that it is zero. However, we have $\mathcal{CP}$ as well as $\mathcal{CPT}$ asymmetries in the medium due to the presence of a nonvanishing electron chemical potential and therefore F_1 is not zero in general[3]. Note also that, in the particular model we have adopted to perform our calculations, the F_1 term does not respect the horizontal $SU(2)_H$ symmetry of the starting Lagrangian (7) and it can be nonvanishing only because the medium, filled up with electrons and not with muons, breaks this symmetry.

The integrals (22) and (23) can be worked out analytically in different regimes

[3]For a detailed discussion on the electromagnetic properties of neutrinos in a medium and the role played by discrete symmetries, see the first ref. in [3].

either for a soft photon, namely when the exchanged momentum q is much smaller than the momenta of the thermalized electrons (of order of T or μ), or for a hard, but almost light-cone photon, $q^2 \to 0$. The calculations for the different regimes, like the ultrarelativistic and the classical ones, can be found in the Appendix. We report here only the result for a degenerate electron gas ($T \ll \mu - m_e$) since they are of interest for the calculations of the plasmon decay rate which will be performed in the next Section. We obtain

$$F_1 = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e \frac{1}{16\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}, \tag{26}$$

$$\begin{aligned} F_2 &= e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e \frac{1}{4\pi^2}\frac{q^2}{m_e^2\Omega^2} \\ &\times \left[\frac{k_F E_F}{2} + m_e^2 \ln\left(\frac{k_F + E_F}{m_e}\right)\right], \end{aligned} \tag{27}$$

where $E_F = \sqrt{m_e^2 + k_F^2} = \mu$ is the Fermi energy.

3.2 Chirality conserving terms

In this Subsection we calculate the contribution from the medium to the chirality conserving term $\nu_e \nu_e \gamma$ (an analogous calculation can be performed for other vertices).

Repeating the same considerations of the Subsection **3.1**, we have to calculate the quantity (see figure 4)

$$\begin{aligned} \mathcal{G}_\mu^{LL} &= e\left|\lambda_{131}\right|^2 \left[\frac{\cos^2\theta_3}{m_{23}^2} + \frac{\sin^2\theta_3}{m_{13}^2}\right] \int \frac{d^4k}{(2\pi)^3}\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right) \\ &\times \left\{\frac{4q^2 \not{k} k_\mu + q^2\left[-2i\sigma_{\mu\nu}q^\nu - 2k_\mu \not{q} + 2\left(k\cdot q\right)\gamma_\mu\right]}{\left(q^2 + 2k\cdot q\right)\left(q^2 - 2k\cdot q\right)}\right. \\ &+ \left.\frac{-2\left(k\cdot q\right)\left[2\not{k} q_\mu - 2k_\mu \not{q} + 2\left(k\cdot q\right)\gamma_\mu\right]}{\left(q^2 + 2k\cdot q\right)\left(q^2 - 2k\cdot q\right)}\right\} L. \end{aligned} \tag{28}$$

Again $q^\mu \mathcal{G}_\mu^{LL} = 0$ for the electromagnetic gauge invariance.

It is then easy to show that the complete electromagnetic vertex function Γ_μ^{LL} can be written under the form

$$\begin{aligned}\Gamma_\mu^{LL} &= \Big[\tilde{F}_1 \tilde{u}_\mu \not{u} + i\tilde{F}_2 \sigma_{\mu\nu} q^\nu \not{u} \\ &+ i\tilde{F}_3 \left(\gamma_\mu u_\nu - \gamma_\nu u_\mu\right) q^\nu \not{u} + \tilde{F}_4 \tilde{g}_{\mu\nu} \gamma^\nu \Big) L,\end{aligned} \tag{29}$$

where

$$\begin{aligned}\tilde{F}_1 &= -\frac{4}{3} e \left|\lambda_{131}\right|^2 \left[\frac{\cos^2\theta_3}{m_{23}^2} + \frac{\sin^2\theta_3}{m_{13}^2}\right] q^2 \int \frac{d^3k}{(2\pi)^3} \frac{(f_- + f_+)}{2E} \\ &\times \frac{\left[4\,(k\cdot u)^2 - m_e^2\right]}{\left(q^2 + 2k\cdot q\right)\left(q^2 - 2k\cdot q\right)},\end{aligned} \tag{30}$$

$$\tilde{F}_2 = F_1, \tag{31}$$

$$\tilde{F}_3 = i\, q^2 \left(q\cdot u\right) \tilde{F}_1, \tag{32}$$

$$\begin{aligned}\tilde{F}_4 &= \frac{8}{3} e \left|\lambda_{131}\right|^2 \left[\frac{\cos^2\theta_3}{m_{23}^2} + \frac{\sin^2\theta_3}{m_{13}^2}\right] q^2 \int \frac{d^3k}{(2\pi)^3} \frac{(f_- + f_+)}{2E} \\ &\times \frac{\left[(k\cdot u)^2 - m_e^2\right]}{\left(q^2 + 2k\cdot q\right)\left(q^2 - 2k\cdot q\right)}.\end{aligned} \tag{33}$$

The expression (29) holds if neutrinos are of the Dirac type. If they are of the Majorana type, then, applying the relation (18), $\Gamma_\mu^{LL,Majorana}$ can be easily obtained by adding to Γ_μ^{LL} its self conjugate term. We then obtain

$$\begin{aligned}\Gamma_\mu^{LL,Majorana} &= -\Big[\tilde{F}_1 \tilde{u}_\mu \not{u} - i\tilde{F}_2 \sigma_{\mu\nu} q^\nu \not{u}\gamma_5 \\ &+ i\tilde{F}_3 \left(\gamma_\mu u_\nu - \gamma_\nu u_\mu\right) q^\nu + \tilde{F}_4 \tilde{g}_{\mu\nu} \gamma^\nu \Big) \gamma_5,\end{aligned} \tag{34}$$

where we have make use of the property $\gamma_5^2 = \mathbf{1}$.

The expressions for the form factors in the different regimes can be found in the Appendix.

The physical interpretation of the form factors can be obtained considering an interaction with an external field. Thus, taking the external field of the form $A^\mu = (\phi, \mathbf{0})$ in the rest frame of the medium, we see that $\tilde{F}_4$ yields an additional

contribution to the charge radius; moreover $\tilde{F}_3$ can be regarded as an additional contribution to the electric dipole moment and $\tilde{F}_2$ to the magnetic one. It is well known that Majorana neutrinos can have neither a charge radius nor diagonal electric or magnetic dipole moments in the vacuum [18] since, for instance, $\bar{\nu}\sigma_{\mu\nu}\nu = 0$ for Majorana neutrinos. Nevertheless, already in the SM they can have electric or magnetic dipole moments in a medium of electrons which introduces $\mathcal{CP}$ as well $\mathcal{CPT}$ asymmetries [3]. We have found that additional contributions can be given by some new physics beyond the SM. These new contributions are again nonvanishing only if the medium is $\mathcal{CP}$ and $\mathcal{CPT}$ asymmetric. Let's take, for instance, the contribution to the magnetic moment. In the non relativistic limit it reduces to

$$\mathcal{O}_M = \tilde{F}_2 \left(\mathbf{s} + \bar{\mathbf{s}}\right) \cdot \mathbf{B}, \tag{35}$$

where $\mathbf{s}$ and $\bar{\mathbf{s}}$ are the spin expectation values for the particles and antiparticles and $\mathbf{B}$ represents a uniform magnetic field. $\mathcal{O}_M$ is odd under both $\mathcal{C}$ and $\mathcal{CPT}$. Since there are strong theoretical reasons to believe that $\mathcal{CPT}$ is conserved by the Lagrangian in the vacuum, any breaking of $\mathcal{CPT}$ must come from the background. Indeed, the particles of the medium must have some chemical potentials associated with them, otherwise $\tilde{F}_2$ must vanish.

We now consider the scattering of an electron neutrino with an external static and uniform magnetic field $\mathbf{B}$. The Dirac equation for a neutrino spinor ψ_ν in the medium can be written as[4]

$$\mathcal{V}\psi_\nu = \left[(1 - a_L)\ \not{k} + (b_L + c_L)\ \not{u}\right]\psi_\nu = 0, \tag{36}$$

where $\mathcal{V}$ is called effective potential. The coefficients a_L and b_L have been

[4] Even in the SM the effective potential receive a correction in presence of a static and uniform magnetic field, see D'Olivo *et al.* in ref. [3], but not proportional to $\not{u}$ as in the class of models considered here.

calculated in [2] whereas

$$
\begin{aligned}
c_L &= \boldsymbol{\mu} \cdot \mathbf{B}, \\
\boldsymbol{\mu} &= i\tilde{F}_2\left(\Omega = 0, |\boldsymbol{Q}| \to 0\right)\boldsymbol{\sigma}.
\end{aligned}
\tag{37}
$$

The value of $\tilde{F}_2$ in the limit $\Omega = 0$, $|\boldsymbol{Q}| \to 0$ has been found, in the non relativistic limit, by D'Olivo *et al.* in ref. [3] and reads

$$
\tilde{F}_2\left(\Omega = 0, |\boldsymbol{Q}| \to 0\right) = \frac{\beta n_-}{4m_e},
\tag{38}
$$

where $n_- = \mathrm{e}^{\beta(\mu - m_e)}\left(2m_e/\beta\right)^{3/2}\left[\Gamma\left(3/2\right)/2\pi^2\right]$ is the number density of electrons. The meaning of eq. (36) is that, in presence of a magnetic field, the effective potential of a neutrino propagating through a medium gets a new contribution proportional to $|\mathbf{B}|$. The relative importance of the matter density effects thus depend on the magnitude of $\mathbf{B}$. For instance, in the sun $|\mathbf{B}|$ is a few tenth of Tesla, the temperature is of order of 1 KeV, so that c_L/b_L is very small, $c_L/b_L \simeq |\lambda_{131}|^2\, 10^{-11}$ for $m_{23} \simeq m_{13} \simeq 100$ GeV. Nevertheless, application to other physical contexts of this new term c_L remains an open question and should be kept in mind.

4. Plasmon decay

It is well known that in a medium composed by a gas of electrons the dispersion relations for transverse and longitudinal photons are quite different from those in the vacuum [9]. Indeed, both modes, called plasmons, acquire an effective plasma mass which allow them to decay into a pair of neutrinos. The process $\gamma_{pl} \to \nu\nu$ represents the primary source for the energy loss of degenerate plasmas, such as red giants and white dwarfs [9] and has recently received

considerable attention [14]. It is well known that already in the SM plasmons can decay into a pair of neutrinos [9]. If neutrinos couple to the photons through a magnetic (or electric) moment in the vacuum, the rate of the energy loss of stellar plasmas due to μ_ν is comparable to the SM contribution for $\mu_\nu \simeq 10^{-12}\mu_B$ and no larger values of μ_ν are tolerated.

In this Section we want to estimate the contribution to the energy loss through the plasmon decay induced by the chirality flipping term proportional to F_1. The differential decay rate of the process $\gamma_{pl}(q) \to \nu_e\,(p_1)\,\nu_\mu\,(p_2)$ due to the F_1 term in the rest frame of the medium (there is no interference contribution with the magnetic moment term) is

$$d\Gamma_{F_1} = \frac{1}{2\Omega}\,(2\pi)^4\,\delta^{(4)}\,(q - p_1 - p_2)\left|\bar{\mathcal{M}}_{F_1}\right|^2 \frac{d^3p_1}{(2\pi)^3\,2E_1}\frac{d^3p_2}{(2\pi)^3\,2E_2}, \tag{39}$$

where

$$\begin{aligned}\left|\bar{\mathcal{M}}_{F_1}\right|^2 &= 4\left(e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right]\, m_e\frac{1}{2\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}\right)^2 \\ &\times \frac{(u\cdot\eta_3)^2}{(\Omega\partial\varepsilon_L/\partial\Omega)}\,(p_1\cdot p_2)\,. \end{aligned} \tag{40}$$

We have neglected neutrino masses and made use if the longitudinal photon vector

$$\eta_3 = \frac{1}{(q^2)^{1/2}}\,(|\boldsymbol{\mathcal{Q}}|\,, 0, 0, \Omega)\,, \tag{41}$$

which satisfies the relation $\eta_3\cdot q = 0$.

In expression (40) ε_L represents the dielectric constant of the longitudinal plasmons [9] and, since we are considering the case of degenerate stars, we are using the expression (26).

Using the Lenard's formula

$$\int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}\,(q - p_1 - p_2)\,p_1^\mu p_2^\nu = \frac{\pi}{24}\left(2p_1^\mu p_2^\nu + g^{\mu\nu}q^2\right), \tag{42}$$

we find that

$$\Gamma_{F_1} = \frac{1}{8\pi}\frac{1}{\Omega^2}\frac{|\boldsymbol{\mathcal{Q}}|^2}{\partial\varepsilon_L/\partial\Omega} \times \left(e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2}-\frac{1}{m_{13}^2}\right]\, m_e\frac{1}{2\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}\right)^2. \qquad (43)$$

The energy loss rate associated to the F_1 term is then given by

$$Q_{F_1} = \int\frac{d^3\boldsymbol{\mathcal{Q}}}{(2\pi)^3}\frac{\Omega\,\Gamma_{F_1}}{\mathrm{e}^{\Omega/T}-1}\left(\varepsilon_L-1\right)^2. \qquad (44)$$

In the case of white dwarfs (red giants before the helium flush) the electron gas is degenerate with a temperature of order of (0.01-0.1) KeV ($\sim$ 8.6 KeV) and the Fermi momentum k_F of order of 495 (400) KeV. Therefore, rigorously speaking, the electron plasma is neither in the nonrelativistic regime nor in the ultrarelativistic one [14]. In such a case the expressions for ε_L and the dispersion relation for longitudinal photons are quite complicated and Q_{F_1} can only be found numerically. Nevertheless, to have an idea of the order of magnitude of Q_{F_1}, we can approximate ε_L and the dispersion relation as

$$\varepsilon_L = 1-\frac{\Omega_0^2}{\Omega^2}\left[1+\frac{3}{5}v_F^2\frac{|\boldsymbol{\mathcal{Q}}|^2}{\Omega^2}\right], \qquad (45)$$

$$q^2 = \Omega_0^2+|\mathcal{Q}|^2\left[\frac{3}{5}v_F^2\frac{\Omega_0^2}{\Omega^2}-1\right], \qquad (46)$$

where $\Omega_0 = (4\alpha k_F^2 v_F/3\pi)^{1/2}$ is the plasma frequency of order 10 KeV for both white dwarfs and red giants and $v_F \sim 0.7$ is the Fermi velocity. Note that, since $\Omega_0 \ll \mu$, the expression (26) of F_1 valid for both soft and hard, but almost light-cone photons, is a good approximation for the physical context under consideration.

With such approximations, Q_{F_1} can be expressed analytically and we find

$$Q_{F_1} \simeq \frac{3}{64\pi^3}\frac{\Omega_0^5}{v_F^5}\left(\frac{5}{3}\right)^{5/2}\frac{\sqrt{2\pi}}{\gamma^{5/2}}\mathrm{e}^{-\gamma}\,\mathcal{K}^2, \qquad (47)$$

where $\gamma = \Omega_0/T$ and we have expressed the coupling constants in terms of the magnetic dipole moment $\mu_{\nu_e\nu_\mu}$

$$\begin{aligned}
\mathcal{K} &= 4\left(\frac{\sin 2\theta_3}{\sin 2\theta_2}\right)\left(\frac{1}{m_{13}^2} - \frac{1}{m_{23}^2}\right)\frac{k_F^3 \mu_{\nu_e\nu_\mu}}{6 m_e m_\tau} \\
&\times \left\{\frac{1}{m_{12}^2}\left[\ln\left(\frac{m_{12}^2}{m^2}\right) - 1\right] - \frac{1}{m_{22}^2}\left[\ln\left(\frac{m_{22}^2}{m^2}\right) - 1\right]\right\}^{-1}.
\end{aligned} \tag{48}$$

In the last expression we have used the fact that the major contribution to $\mu_{\nu_e\nu_\mu}$ comes from the diagram 2a) and 2b). In a similar way one can find the energy loss rate due to the decay $\gamma_{pl} \to \nu_e\nu_\mu$ through the magnetic dipole moment. In the range of interest of temperatures and densities the longitudinal and transverse contributions are comparable and, for instance,

$$Q_{long} \simeq \frac{1}{12}\frac{\mu_{\nu_e\nu_\mu}^2}{(2\pi)^3}\left(\frac{5}{3}\right)^{3/2}\frac{\Omega_0^7}{v_F^3}\sqrt{2\pi}\gamma^{-3/2}\mathrm{e}^{-\gamma}. \tag{49}$$

The ratio between Q_{F_1} and Q_{long} is then

$$\begin{aligned}
\frac{Q_{F_1}}{Q_{long}} &\simeq \frac{15}{2}\frac{1}{\mu_{\nu_e\nu_\mu}^2}\frac{\mathcal{K}^2}{v_F^2\,\Omega_0^2\,\gamma} \\
&\simeq 10^{-5}\left(\frac{0.7}{v_F}\right)^2\left(\frac{10\,\mathrm{KeV}}{\Omega_0}\right)^2\left(\frac{10}{\gamma}\right)\left(\frac{k_F}{400\,\mathrm{KeV}}\right)^6 \\
&\times \left(\frac{\sin 2\theta_3}{\sin 2\theta_2}\frac{m_{13}^2 - m_{23}^2}{m_{12}^2 - m_{22}^2}\right)^2\left(\frac{m_2}{m_3}\right)^8,
\end{aligned} \tag{50}$$

where we have indicated with m_k the averaged eigenvalue of the mixing matrix $\tilde{l}_L^k\tilde{l}_R^k$. Since in supersymmetric models the factor $(\sin 2\theta_3/\sin 2\theta_2)\left(m_{13}^2 - m_{23}^2/m_{12}^2 - m_{22}^2\right)^2$ is of order of (m_τ/m_μ) [17], one can obtain, $Q_{F_1} \simeq Q_{long}$ for $m_2 \simeq 2m_3$. Even if the above estimation is approximate and holds in the particular framework we have chosen, the general message one can read from it is that, going beyond the SM, one has to take into account all the possible terms which arise at finite temperature and density for the $\nu\nu\gamma$ vertex because the new terms can give non negligible contributions to relevant processes as the plasmon decay in degenerate stars.

5. Conclusions

In the present work we have carried out an explicit calculation of the neutrino electromagnetic vertex in a background of electrons in a large class of models where charged scalar bosons couple to leptons. We have been motivated by the fact that such models are able to provide a magnetic moment as large as $\mu_\nu \sim 10^{-12}\, \mu_B$, which can play a relevant role in different astrophysical phenomena.

We have shown that the contribution from the medium to the magnetic (electric) dipole moment is not significant, but a new chirality flipping, but helicity conserving term, arises. This new term is associated to the longitudinal photons and therefore disappears in the vacuum and can be nonvanishing only because the medium does not respect $\mathcal{CPT}$. We have also estimated the contribution of this new term to the plasmon decay rate showing that it can be comparable with the contribution coming from the vacuum magnetic moment. Therefore it must be taken into account in different applications of the vertex $\nu\nu\gamma$ in a medium. Finally, we have calculated the correction to the effective potential of a propagating neutrino in presence of a magnetic field. Although the application of this to the solar neutrino puzzle seems to be uninteresting, the possible applications in other contexts deserve further consideration and are currently under study.

Acknowledgments

It is a pleasure to thank K. Enqvist and A. Masiero for reading the early version of the paper and for useful suggestions.

Appendix.

In this Appendix we give some details and the complete results for the form factors introduced in the text for different regimes in the limit of soft photons, namely when the exchanged momentum q is much smaller than the momenta of the thermalized electrons, or for a hard, but almost light-like photon, $q^2 \to 0$.

Chirality flipping terms

Ultrarelativistic regime $(T, \mu \gg m_e)$

$$\begin{aligned} F_1 &= e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{13}^2} - \frac{1}{m_{23}^2}\right] \\ &\times \frac{1}{8\pi^2\beta}\left[a\left(m_e\beta, -\mu\right) - a\left(m_e\beta, +\mu\right)\right], \end{aligned} \tag{A. 1}$$

where

$$a\left(m_e\beta, \pm\mu\right) = \ln\left[1 + \mathrm{e}^{-(m_e \pm \mu)\beta}\right], \quad T > \mu, \tag{A. 2}$$

and

$$a\left(m_e\beta, -\mu\right) \simeq \mu - m_e, \quad T < \mu; \tag{A. 3}$$

$$\begin{aligned} F_2 &= -e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \left[\frac{1}{m_{13}^2} - \frac{1}{m_{23}^2}\right] \\ &\times \frac{1}{4\pi^2}\left[b\left(m_e\beta, -\mu\right) + b\left(m_e\beta, +\mu\right)\right], \end{aligned} \tag{A. 4}$$

where

$$b\left(m_e\beta, \pm\mu\right) = \sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{\mp n\beta\mu}\mathrm{Ei}\left(-n\beta m_e\right), \quad T > \mu, \tag{A. 5}$$

and

$$b\left(m_e\beta, -\mu\right) = \ln\left(\mu/m_e\right), \quad T < \mu, \tag{A. 6}$$

where $\mathrm{Ei}(x)$ is the exponential-integral function.

Classical limit ($T \ll m_e$ **and** $(m_e - \mu) \gg T$)

$$
\begin{aligned}
F_1 &= -\frac{1}{8} e \frac{\lambda_{123}\lambda_{131}}{2} \sin 2\theta_3 \left[\frac{1}{m_{13}^2} - \frac{1}{m_{23}^2}\right] \\
&\times \; q^2 \frac{\sqrt{\pi}}{\Gamma(3/2)} \frac{n_-}{m_e^2 \Omega^2}, \qquad \text{(A. 7)} \\
F_2 &= \frac{1}{4} e \frac{\lambda_{123}\lambda_{131}}{2} \sin 2\theta_3 \left[\frac{1}{m_{13}^2} - \frac{1}{m_{23}^2}\right] \\
&\times \; q^2 \frac{\sqrt{\pi}}{\Gamma(3/2)} \frac{\beta n_-}{m_e^2 \Omega^2}, \qquad \text{(A. 8)}
\end{aligned}
$$

where $n_- = \mathrm{e}^{\beta(\mu - m_e)} \left(2m_e/\beta\right)^{3/2} \left[\Gamma\left(3/2\right)/2\pi^2\right]$.

Chirality conserving terms

To calculate the chirality conserving form factors, we must calculate the following integral

$$
I_{\lambda\nu} = \int \frac{d^4k}{(2\pi)^3} \frac{\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right)}{\left(q^2 + 2q\cdot k\right)\left(q^2 - 2q\cdot k\right)} k_\lambda k_\nu. \qquad \text{(A. 9)}
$$

Since $I_{\lambda\nu} = I_{\nu\lambda}$ and $I_{\lambda\nu}(q) = I_{\nu\lambda}(-q)$, $I_{\lambda\nu}$ must be of the form

$$
I_{\lambda\nu} = A u_\lambda u_\nu + B g_{\lambda\nu} + C q_\lambda q_\nu. \qquad \text{(A. 10)}
$$

If we then contract $I_{\lambda\nu}$ with u^λ the result must be proportional to u_ν, from which we read that $C = 0$. From eq. (A.9) we have

$$
\begin{aligned}
I_1 &= u_\lambda u_\nu I^{\lambda\nu} = A + B = \int \frac{d^4k}{(2\pi)^3} \frac{\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right)\left(k\cdot u\right)^2}{\left(q^2 + 2q\cdot k\right)\left(q^2 - 2q\cdot k\right)}, \qquad \text{(A. 11)} \\
I_2 &= g_{\lambda\nu} I^{\lambda\nu} = A + 4B = \int \frac{d^4k}{(2\pi)^3} \frac{\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right) m_e^2}{\left(q^2 + 2q\cdot k\right)\left(q^2 - 2q\cdot k\right)}. \qquad \text{(A. 12)}
\end{aligned}
$$

The chirality conserving form factors are then functions of I_1 and I_2

$$
\tilde{F}_1 = \frac{1}{3}\left(4I_1 - I_2\right),
$$

$$\begin{aligned}
\tilde{F}_2 &= -2F_1, \\
\tilde{F}_3 &= -iq^2\tilde{F}_1, \\
\tilde{F}_4 &= \frac{1}{3}\left(I_2 - I_1\right). \qquad \text{(A. 13)}
\end{aligned}$$

Ultrarelativistic regime $(T, \mu \gg m_e)$

$$I_1 = \frac{-1}{16\pi^2}\frac{1}{q^2\beta^2}\left[c\left(m_e\beta, +\mu\right) + c\left(m_e\beta, -\mu\right)\right], \qquad \text{(A. 14)}$$

$$I_2 = \frac{-m_e^2}{16\pi^2 q^2}\left[b\left(m_e\beta, +\mu\right) + b\left(m_e\beta, -\mu\right)\right], \qquad \text{(A. 15)}$$

where

$$c\left(m_e\beta, \pm\mu\right) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\mathrm{e}^{-n\beta(m\pm\mu)}, \quad T > \mu, \qquad \text{(A. 16)}$$

and

$$c\left(m_e\beta, -\mu\right) = \frac{\mu^2}{2}, \qquad T < \mu. \qquad \text{(A. 17)}$$

Degenerate limit $(T \ll m_e$ **and** $(\mu - m_e) \gg T)$

$$I_1 = -\frac{1}{16\pi^2}\frac{1}{\Omega^2 m_e^2}\left[\frac{E_F^3 k_F}{4} - \frac{E_F m_e k_F}{8} - \frac{m_e^3}{8}\ln\left(\frac{E_F + k_F}{m_e}\right)\right], \qquad \text{(A. 18)}$$

$$I_2 = -\frac{1}{16\pi^2}\frac{1}{\Omega^2}\left[\frac{E_F k_F}{2} - \frac{m_e^2}{2}\ln\left(\frac{E_F + k_F}{m_e}\right)\right]. \qquad \text{(A. 19)}$$

Classical limit $(T \ll m_e$ **and** $(m_e - \mu) \gg T)$

$$I_1 \simeq I_2 = -\frac{1}{16}\frac{1}{m_e^2\Omega^2}\frac{n_-\sqrt{\pi}}{\Gamma(3/2)}. \qquad \text{(A. 20)}$$

References

[1] L. Wolfenstein, Phys. Rev. **D17** (1978) 2369; P. Langacker, J.P. Leveille and J. Sheiman, Phys. Rev. **D27** (1983) 1228.

[2] V.N. Oraevsky and V.B. Semikoz, Physica **A142** (1987) 135; V.B. Semikoz and Y.A. Smorodinskii, JETP **68** (1989) 20; D. Nötzold and G.G. Raffelt, Nucl. Phys. **B307** (1988) 924; P.B. Pal and T.N. Pham, Phys. rev. **D40** (1989) 259; J.F. Nieves, Phys. Rev. **D40** (1989) 866; K. Enqvist, K. Kainulainen and J. Maalampi, Nucl. Phys. **B349** (1991) 754; J.C. D'Olivo, J.F. Nieves and M. Torres, Phys. Rev. **D46** (1992) 1172.

[3] J.F. Nieves and P.B. Pal, Phys. Rev. **D40** (1989) 1963; J.C. D'Olivo, J.F. Nieves and P.B. Pal, Phys. Rev. **D40** (1989) 3679; V.N. Oraevsky, V.B. Semikoz and Ya.A. Smorodinsky, Phys. Lett. **227B** (1989) 255; C. Giunti, C.W. Kim and W.P. Lam, Phys. Rev **D43** (1991) 164; T. Altherr and K. Kainulainen, Phys. Lett. **B262** (1991) 79; S.S. Massod, Phys. Rev. **D48** (1993) 3250; J.F. Nieves and P.B. Pal, LTP-058-UPR preprint (1993).

[4] J.C. D'Olivo, J.F. Nieves and P.B. Pal, Phys. Rev. Lett. **64** (1990) 1088.

[5] For references and a recent review, see P.B. Pal, Int. Journ. Mod. Phys. **A7** (1992) 5387.

[6] S.P. Mikheyev, A.Y. Smirnov, Nuovo Cimento **C9** (1986) 17.

[7] M.B. Voloshin, M.I. Vysotsky and L.B. Okun, Sov. J. Nucl. Phys. **44** (1986) 440; Sov. Phys. JETP **64** (1986) 446; M.B. Voloshin and M.I. Vysotsky, Sov. J. Nucl. Phys. **44** (1986) 544; L.B. Okun, Sov. J. Nucl.

Phys. **44** (1986) 546; R. Barbieri and G. Fiorentini, Nucl. Phys. **B304** (1989) 909.

[8] C.S. Lim and W.J. Marciano, Phys. Rev. **D37** (1988) 1368; E.W. Akhmedov, Phys. Lett. **B213** (1988) 64.

[9] See, for instance, G.G. Raffelt, Phys. Rep. **198** (1990) 1 and references therein.

[10] J. Morgan, Phys. Lett. **B102** (1981) 247.

[11] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311.

[12] See, for instance, A. Zee, Phys. Lett. **B93** (1980) 389; M. Fukugita and T.B. Yanagida, Phys. Rev. lett. **58** (1987) 1807; K.S. Babu and V.S. Mathur, Phys. Rev. Lett. **196** (1987) 218; M.B. Voloshin, Sov. J. Nucl. Phys. **48** (1988) 512; R. Barbieri and R.N. Mohapatra, Phys. Lett. **B218** (1988) 225; G. Ecker, W. Grimus and H. Neufeld, Phys. Lett. **B232** (1989) 217; D. Chang, W. Keung and G. Senjanovic, Phys. Rev. **D42** (1990) 1599.

[13] R. Barbieri, M.M. Guzzo, A. Masiero and D. Tommasini, Phys. lett. **B252** (1990) 251; K.S. Babu and R.N. Mohapatra, Phys. Rev. lett. **64** (1990) 1705.

[14] E. Braaten, Phys. Rev. Lett. **66** (1991) 1655; E. Braaten and D. Segel, Phys. Rev. **D47** (1993) 1478; R.F. Sawyer, Phys. Rev. **D46** (1992) 1180; N. Itoh, H. Mutoh, A. Hikita and Y. Kohyama, Ap. J. **395** (1992) 622; M. Haft, G.G. Raffelt and A. Weiss, to appear in Ap. J.; T. Altherr, E. Petitgirard and T. del Riogaztelurrutia, Astr. Part. Phys. **1** (1993) 289; T. Altherr, CERN preprint (1993).

[15] C.S. Aulakh and R.N. Mohapatra, Phys. Rev. **D119** (1983) 136; L.J. Hall and M. Suzuki, Nuc. Phys. **B231** (1984) 419; I.H. Lee, Nuc. Phys. **B246** (1984) 120; J. Ellis et al., Phys. Lett. **150** (1985) 142; G.G. Ross and J.W. Valle, Phys. Lett. **B151** (1985) 375; S. Dawson, Nuc. Phys. **B261** (1985) 297; R.N. Mohapatra, Phys. Rev. **D11** (1986) 3457; E. Ma and P. Roy, Phys. Rev. **D41** (1990) 988; E. Ma and D. Ng, Phys. Rev. **D41** (1990) 1005; S. Dimopoulos, R. Esmailzadeh, L.J. Hall, J.P. Merlo and G.S. Starkmen, Phys. Rev. **D41** (1990) 2099; H. Dreiner and R.J.N. Phillips, Nuc. Phys. **B367** (1991) 591; L.E. Ibàñez and G.G. Ross, Nuc. Phys. **B368** (1992) 3; K. Enqvist, A. Masiero and A. Riotto, Nucl. Phys. **B373** (1992) 95.

[16] We want to stress that, in the case of light Dirac neutrinos, the strength of the coupling of these new charged scalar bosons to the leptons is constrained by the requirement that the right-handed components decouple before the quark-hadron phase transition so that their density can be diluited by the subsequent releasing of entropy and give no appreciable contribution to the density of the Universe during at the onset of the primordial nucleosynthesis. If G_F^R is the strength of the above-mentioned coupling, then G_F^R must be less that $\sim 10^{-3}\, G_F$ (see A.I. Rez and V.B. Semikoz, Phys. lett. **B249** (1990) 125). Again, the induced limit on μ_ν is of order of $10^{-12}\, \mu_B$.

[17] For a review, see for example, H. Nilles, Phys. Rep. **110** (1984) 1 and references therein.

[18] J.F. Nieves, Phys. Rev. **D26** (1982) 3152.

Figure Caption

Fig. 1 Feynman diagrams contributing to $\mu_{\nu_i\nu_j}$ after a photon insertion line in any charged internal line.

Fig. 2 Feynman diagrams contributiong to $\mu_{\nu_e\nu_\mu}$ (after a photon insertion line in any charged internal line) when $L_e - L_\mu$ conservation is imposed.

Fig. 3 Relevant Feynman diagram for the $\nu_e\nu_\mu\gamma$ vertex in a background of electrons.

Fig. 4 Relevant Feynman diagram for the $\nu_e\nu_e\gamma$ vertex in a background of electrons.